VARS-TOOL

Advances in handling correlation effects between model parameters

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KEY POINTS

- Extend the theory of VARS to handle non-uniform and correlated inputs
- Develop gSTAR-VARS to sample any joint and conditional probability distributions
- Show properly accounting for correlation effects, which are often ignored, is essential in sensitivity analysis.

VARS FRAMEWORK

$$\gamma(h_{x_i}) = \frac{1}{2}E\left\{\left[f(x_i + h_{x_i}|x_{\sim i}, x_{\sim i}) - f(x_i|x_{\sim i}, x_{\sim i})\right]^2\right\}$$

$$\gamma(h_{x_i}) = \frac{1}{2N(h_{x_i})}\sum\left[f(x_i + h_{x_i}|x_{\sim i}, x_{\sim i}) - f(x_i|x_{\sim i}, x_{\sim i})\right]^2$$

CORRELATION EFFECTS

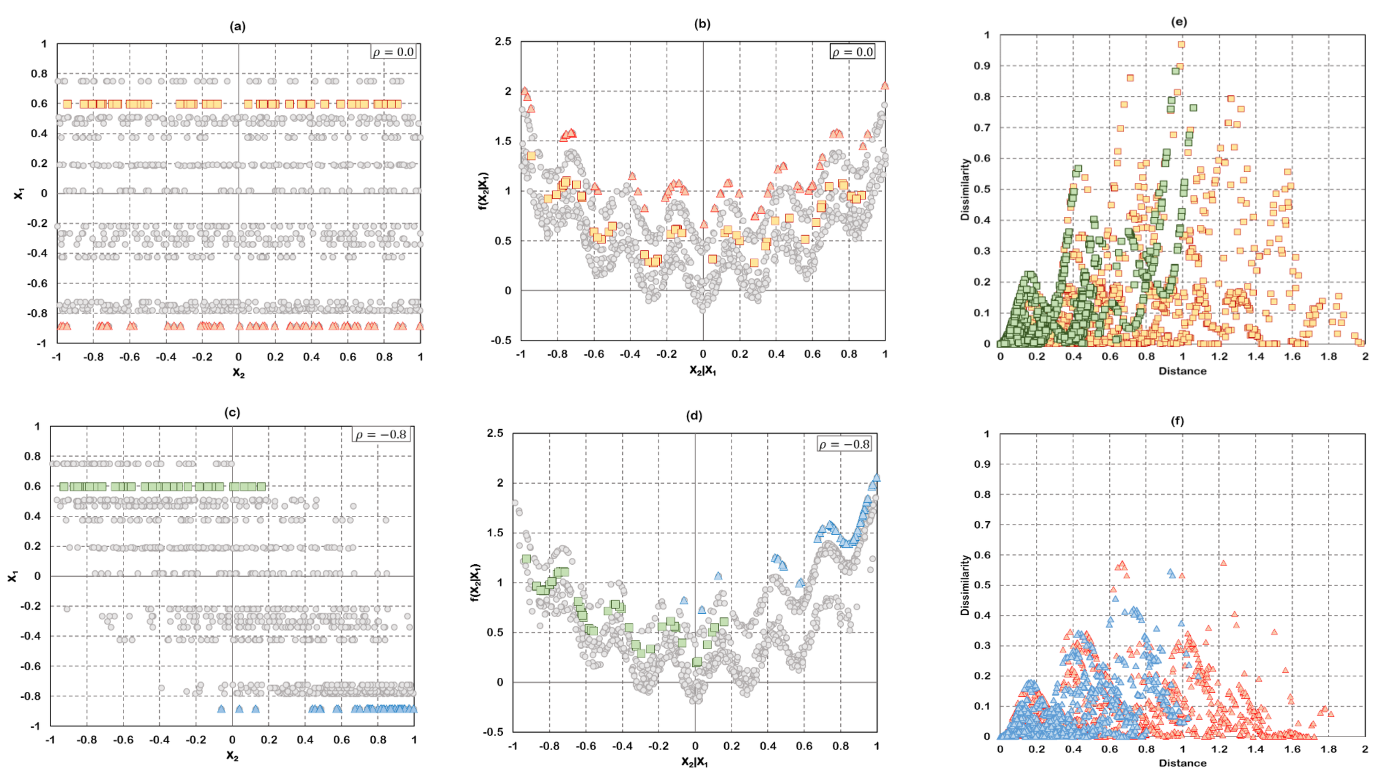


Figure 2: Correlation effects of model parameters on variogram structures

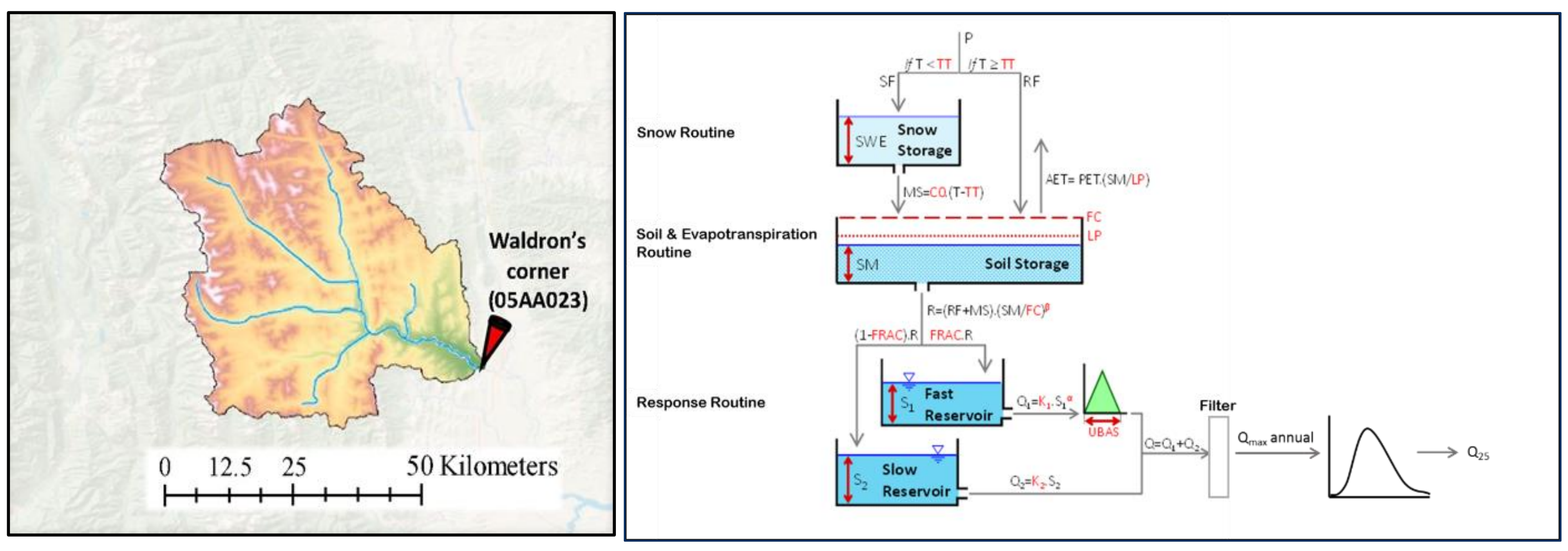


Figure 4: Oldman River Watershed and the HBV-SASK model for flood frequency analysis

Table 2: Projected temperature and precipitation change from 2016-2035 over Alberta, Canada based on three RCPs

Season	Change in temperature (°C)			Change in precipitation (%)		
	RCP2.6	RCP4.5	RCP8.5	RCP2.6	RCP4.5	RCP8.5
Winter (Dec-Feb)	1.2	1.2	1.8	3.1	5.9	4.3
Summer (Jun-Aug)	1.1	1.1	1.4	3.3	2.3	1.7

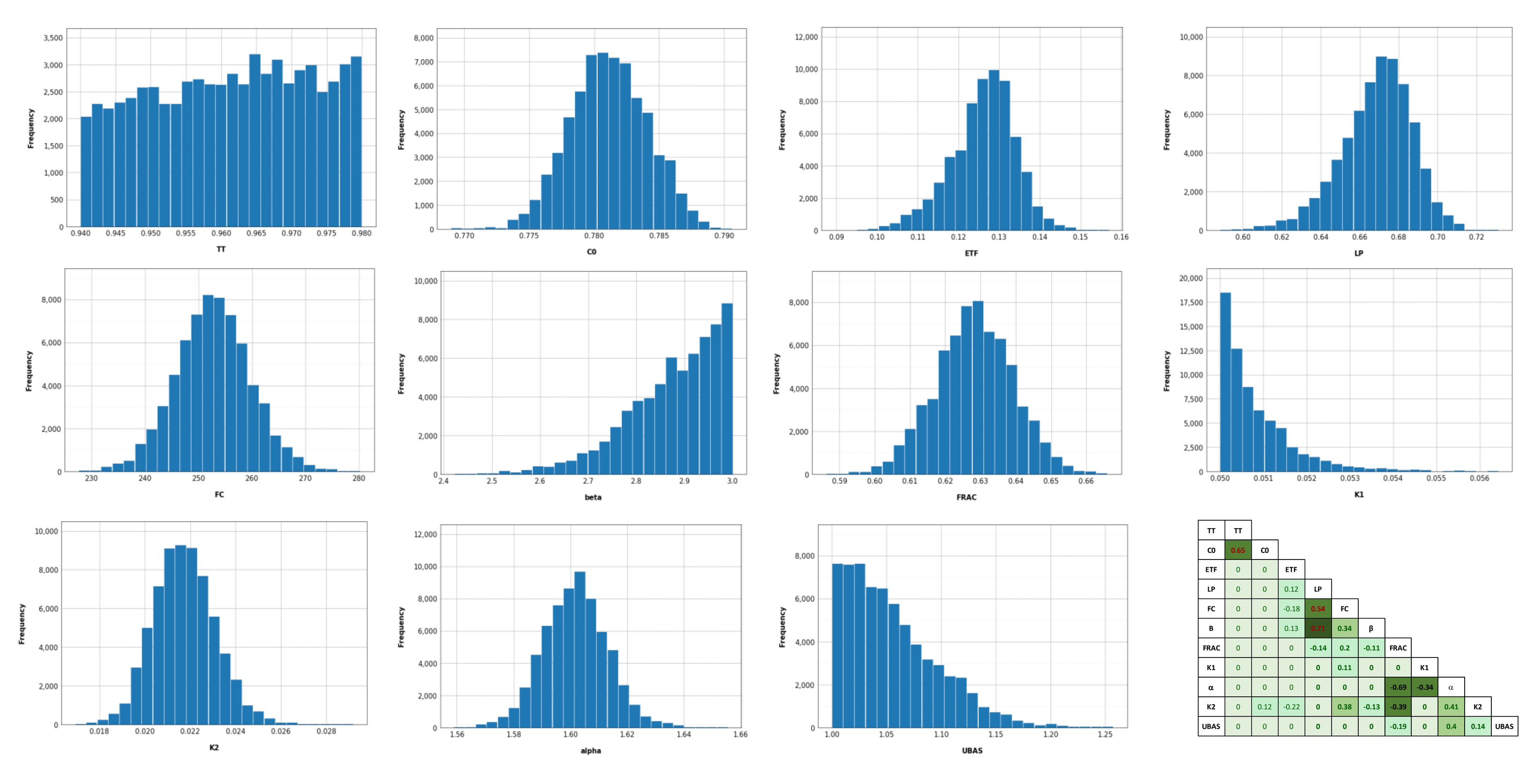
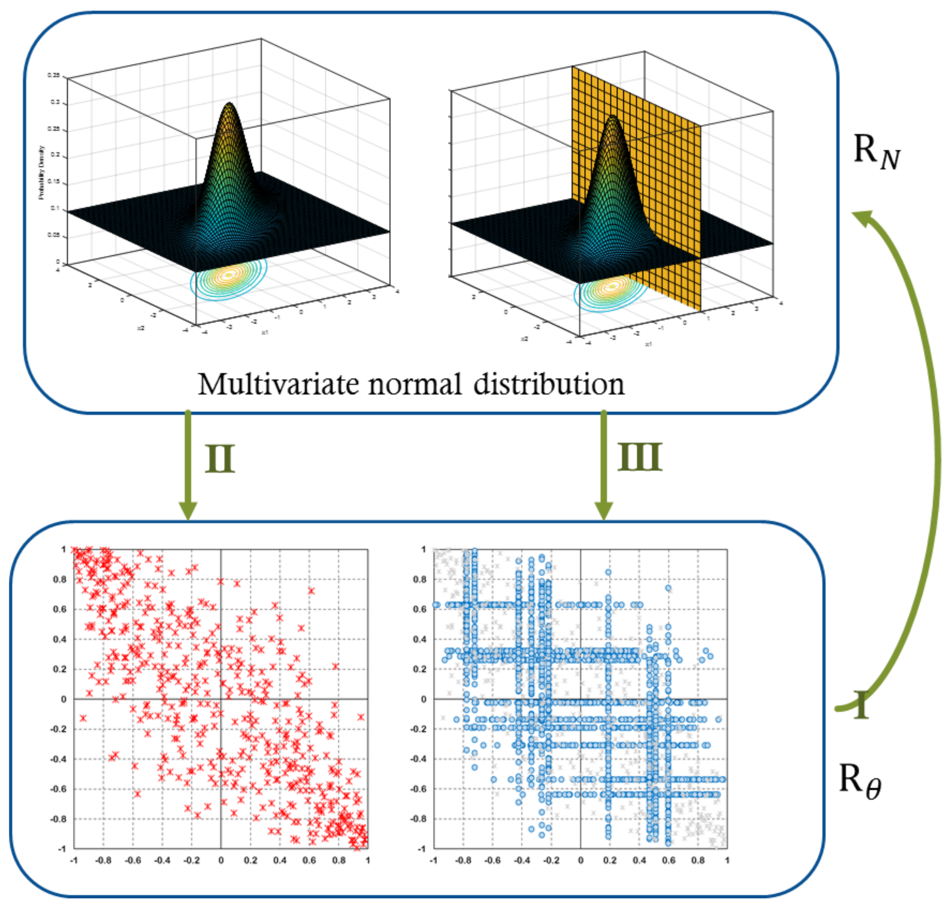


Table 5: Histograms of the inferred parameter values a posteriori by the MCMC algorithm

GENERALIZED STAR-BASED SAMPLING PROCESS



Process:

I. Mapping the correlation matrix in "actual space" to "Normal space"

$$\rho_{x_{i,j}} = \frac{1}{\sigma_{x_i}\sigma_{x_j}}\iint_{R^2}\left\{\left[F_{x_i}^{-1}[\Phi(z_i)] - \mu_{x_i}\right]\left[F_{x_j}^{-1}[\Phi(z_j)] - \mu_{x_j}\right]\varphi_2\left(z_i, z_j, \rho_{z_{i,j}}\right)dz_i dz_j\right\}$$

II. Sample from the multivariate standard normal distribution then transform to the designated multivariate distribution

III. Sample from conditional normal distributions then transform to their corresponding designated conditional distributions.

Figure 1: Generalized star-based sampling process

APPLICATION

Identify Controlling factors of flood estimates under future climate changes

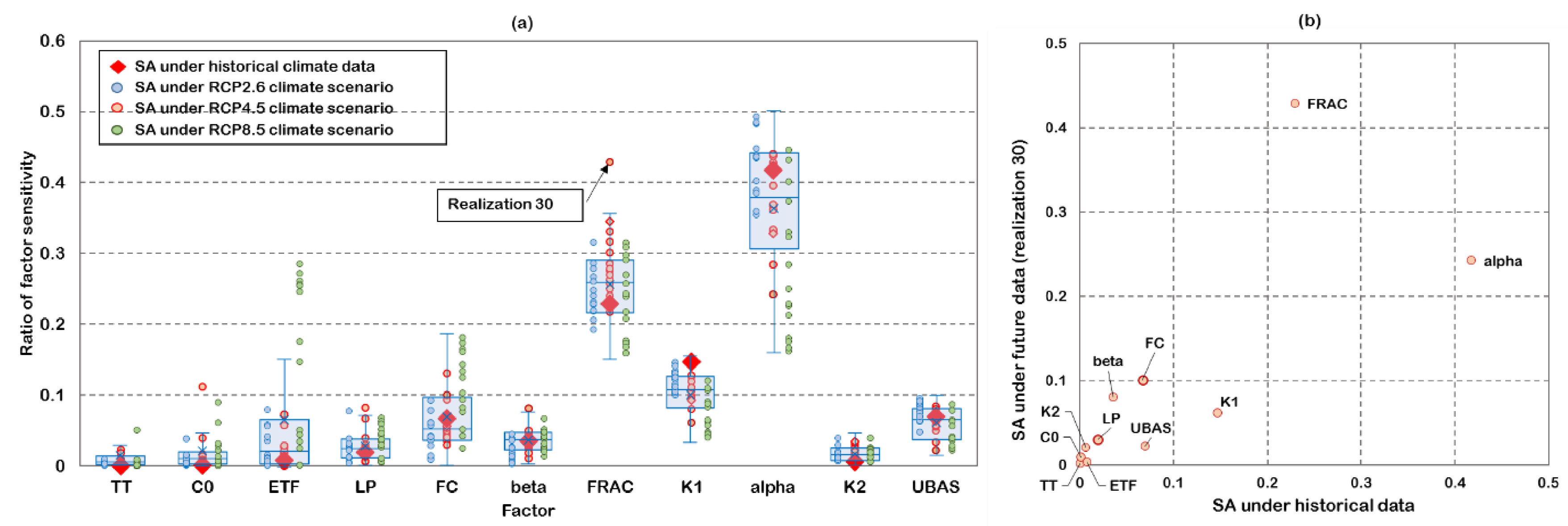


Figure 3: (a) Sensitivity analysis of HBV-SASK model under plausible future scenarios and (b) Comparison between sensitivity analysis using historical climate data and sensitivity analysis using a specific future realization (future scenario 30)

Table 1: HBV-SASK model parameters and their initial ranges

No.	Parameters	Name	Lower Limit	Upper Limit
I. Snow routine				
1	TT	Air temperature threshold in °C for melting/freezing and separating rain and snow.	-4	4
2	C0	Base melt factor, in mm/°C per day.	0	10
II. Soil and evapotranspiration routine				
3	ETF	Temperature anomaly correction in 1/°C of potential evapotranspiration.	0	1
4	LP	Limit for daily potential evapotranspiration as a multiplier to the field capacity of soil (FC)	0	1
5	FC	Field capacity of soil, in mm. The maximum amount of water that the soil can retain.	50	500
6	β	Shape parameter (exponent) for soil release equation	1	3
III. Response routine				
7	FRAC	Fraction of soil release entering fast reservoir.	0.1	0.9
8	K1	Fast reservoir coefficient, which determines proportion of the storage being released per day.	0.05	1
9	α	Shape parameter (exponent) for fast reservoir equation.	1	3
10	K2	Slow reservoir coefficient, which determines proportion of the storage being released per day.	0	0.05
11	UBAS	Base of unit hydrograph for watershed routing in day; default is 1 for small watersheds.	1	3

CONCLUSIONS

A novel approach for the GSA of models with correlated, non-uniformly distributed variables is introduced.

The proposed approach is an extension of the theory of Variogram Analysis of Response Surfaces (VARS).

Different sensitivity indices, including the integrated variograms ($IVARS_{10}$, $IVARS_{30}$, and $IVARS_{50}$) and the variance-based total-order effects (VARS-TO), obtained from the proposed method can provide a comprehensive characterization of sensitivity across the full spectrum of perturbation scales in the factor space.